1. **Symmetric matrices.** Show that for any matrix $\mathbf{A} \in \mathbb{R}^{n \times m}$ matrices $\mathbf{A}\mathbf{A}^T$ and $\mathbf{A}^T\mathbf{A}$ are symmetric.

2. **Postive semidefinite matrices.** Show that for any matrix $\mathbf{A} \in \mathbb{R}^{n \times m}$ matrices $\mathbf{A}\mathbf{A}^T$ and $\mathbf{A}^T\mathbf{A}$ are postive semidefinite.

3. **Postive definite matrices.** Show that for any matrix $\mathbf{A} \in \mathbb{R}^{n \times m}$ with independent rows and columns matrices $\mathbf{A}\mathbf{A}^T$ and $\mathbf{A}^T\mathbf{A}$ are postive definite.

4. **Eigenvalues and eigenvectors.** Find the eigenvalues and eigenvectors of the following matrix. Normalize the eigenvectors to unit vectors.
$$\mathbf{A} = \begin{pmatrix} 3 & -7 \\ 1 & -5 \end{pmatrix}$$

5. **Linear independence.** Let λ_1, λ_2, …, λ_n be distinct eigenvalues of a matrix from $\mathbb{R}^{n \times n}$. Show that the corresponding eigenvectors $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_n$ form a linearly independent set.

6. **Independence.** Let $\Omega = \{1, 2, 3, 4\}$ be a state (atomic event) space, and $\mathcal{A} = 2^\Omega$ be a set of all possible events. Let $P(i) = \frac{1}{4}$, where $i = 1, 2, 3, 4$ be the probability of an atomic event taking place and let $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{2, 3\}$ be three events.

   a) Are A, B, and C pairwise independent?

   b) Are A and B conditionally independent given C?

7. **Independence and covariance.** Let X and Y be two independent discrete random variables:

   a) Show that the expectation of the product of XY equals to the product of their individual expectations:
   $$E[XY] = E[X]E[Y]. \tag{1}$$

   b) Show that if X and Y are independent then their covariance $cov(X, Y) = 0$.

8. **Bayes Theorem.**: Suppose A and B are events such that $P(A) > 0$ and $P(B) > 0$.

   a) State the Bayes theorem.

   b) Prove the Bayes theorem.

9. **Sample mean.** Suppose $X_1, \dots, X_n$ are independent and identical r.v. with the expectation μ and variance σ^2. Let S_n be the n-th sample average of X_i: $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. How is S_n distributed for large n?

10. **Maximum Likelihood Estimation.** We make m experiments with n coin flips each. Let $X_1, \dots, X_m$ be r.v. denoting number of heads in each experiment. We can model X_i as a binomial r.v. with parameters (p, n). Estimate p with the method of maximum likelihood.