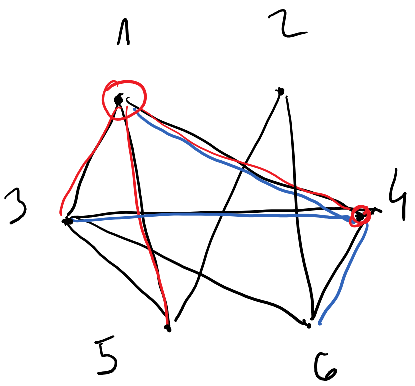
Freitag, 5. Oktober 2018 09:07

The adjacency matrix

Ex. 1: Undirected simple graph



7

$$A = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ & & & & & \\ & & & & & \\ 1 & 0 & 1 & 0 & 0 & 1 \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}$$

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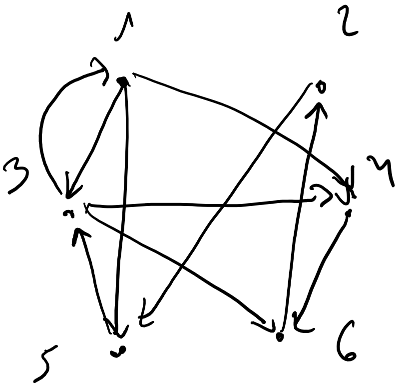
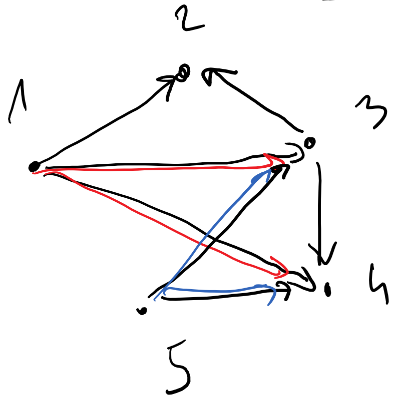
Ex 2. Simple directed graph



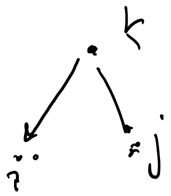
$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & & & & & \\ 1 & & & & & \\ 1 & & & & & \\ 1 & & & & & \\ 0 & & & & & \end{bmatrix} \end{array}$$

Cocitation



$$A = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\Sigma \; 1 \; 0 \; 0 \; 0 \; 1 = 2$



$\underset{1}{\underline{A_{ik}}} \;\&\; \underset{1}{\underline{A_{jk}}} \iff A_{ik} \cdot A_{jk} = 1$

$A \cdot B$

$C = AA^T$

C_{ij} = cocitation betw i and j

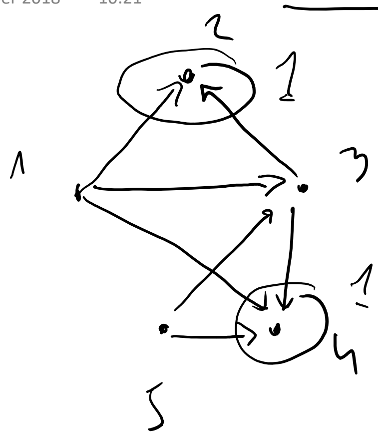
Symmetry: $M = M^T$

$C = AA^T$

$\underline{AA^T}$

$(AA^T)^T = A^{T^T} A^T = \underline{AA^T}$

Bib. coupling



$$A = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{ccccc|l} 1 & 2 & 3 & 4 & 5 & \text{outgoing} \end{array}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}$$

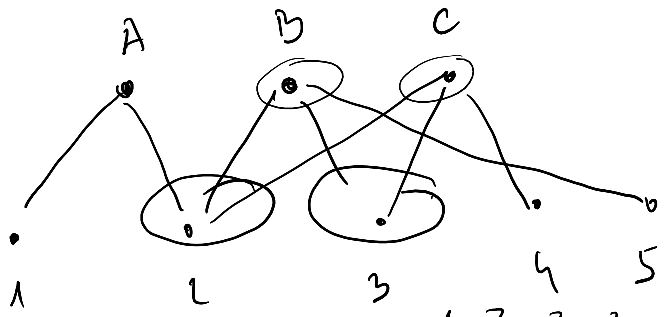
nodes i, j

Bib. coupling

→ $B = A^T A$

$$\begin{array}{ccc} A_{ki} & A_{kj} & \Sigma \\ \| & \| & 0 \\ 1 & 1 & 1 \\ & & 0 \\ & & 1 \\ B_{ij} & & 0 \end{array} \Bigg) = 2$$

Incidence matrix: Bipartite networks



$$B = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}$$
$$B = \left[\begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right]\begin{array}{c} A \\ B \leftarrow \\ C \leftarrow \end{array}$$

$A A^T$

$A^T A$ $\quad B B^T$

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ A \\ B \\ C \end{array}\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & A & B & C \end{array}$$
$$A = \left[\begin{array}{c|c} \cancel{0} & B^T \\ \hline B & \cancel{0} \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ A \\ B \\ C \end{array}$$

Projections on groups and on nodes

${}_{3\times5}B B^T{}_{5\times3}$ groups

$B^T B$ nodes

Problems

$A \in \mathbb{R}^{n\times n}$ $\quad \vec{1} \in \mathbb{R}^{n} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ $\quad \vec{1}^{T} = [\underbrace{1\ 1\ \dots\ 1}_{n\text{-times}}]$

$\vec{k} = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$

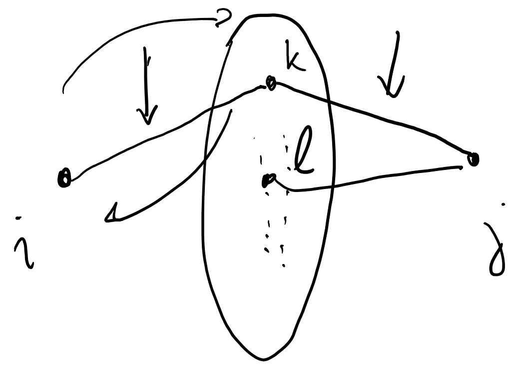
1) $\vec{k} = A\vec{1}$

2) $= \frac{1}{2}\vec{1}^{T}\vec{k} = \frac{1}{2}\vec{1}^{T}A\vec{1}$

$$\underset{n\times n}{A}\ \underset{n\times 1}{x} = \underset{n\times 1}{y}$$

3) Number of paths of length 2 between i, j

$\boxed{A}$



path of len 4

A^2

$A_{ik} \cdot A_{kj}$ $\quad A_{ik} A_{kj} = 1$ iff (i,k) (k,j)

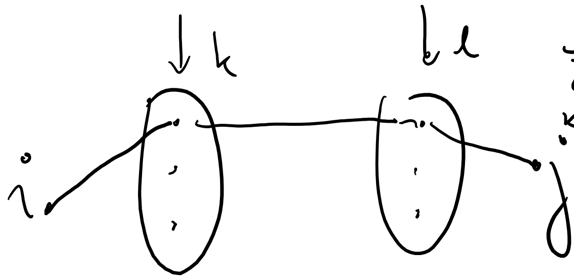
$\parallel \qquad \parallel$

$1 \cdot 1$ $\quad A_{il} A_{lj} = 0$

$A_{il} = 0 \quad A_{lj} = 1$

$\sum_k A_{ik} A_{kj}$

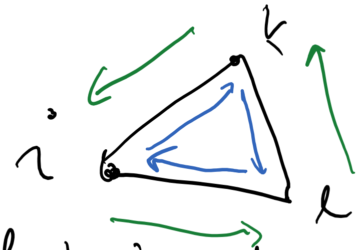
4) Number of paths of length 3

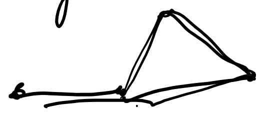


$$\sum_{k,l} A_{ik} A_{kl} A_{lj}$$

$$\boxed{A^3 = A \cdot A \cdot A}$$

$\rightarrow$ triangles





5) Number of triangles $\frac{1}{6} \mathrm{Tr}(A^3)$

$\mathrm{Tr}(A^r) \leftarrow$

Eigenvalues and eigenvectors

Literature

Gilbert Strang:

Introduction to LA

Computational science and engineering

Carl Meyer:

Matrix Analysis and Applied LA

Eigenvalues and eigenvectors

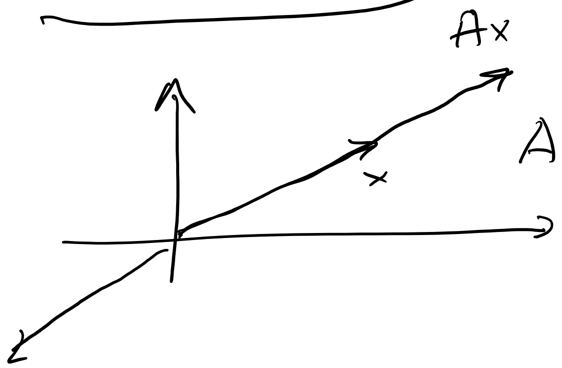
$A \in \mathbb{R}^{n \times n}$ $\lambda \in \mathbb{C}$ $x \in \mathbb{C}^n$ $x \neq \vec{0}$

$\lambda = a + ib$, $i = \sqrt{-1}$

$$\boxed{Ax = \lambda x}$$

x is eigenvector

λ corr. eigenvalue



Scaling property of eigenvectors

x $\qquad$ $c \in \mathbb{R}$

$y = c \cdot x$

cx is also eig[...]

λ

$$\underline{A(cx)} = \lambda_e (cx)$$

$$cAx = c \cdot \lambda \cdot x = \underline{\lambda \cdot (cx)}$$

$$e_x = \frac{x}{\|x\|_2} = \frac{x}{\sqrt{x'x}} \qquad \|e_x\| = 1$$