Diffusion

$j \to i \quad \Delta u = \underline{D(u_j - u_i)\,\Delta t} \qquad u \in \mathbb{R}^n$

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$$\frac{\Delta u}{\Delta t} = D(u_j - u_i)$$

$$\lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t} = \frac{du}{dt} = D(u_j - u_i)$$
$$= C(u_j - u_i)$$

a) $$\frac{du_i}{dt} = C \sum_j A_{ij} (u_j - u_i) = C\left(\sum_j A_{ij} u_j - u_i \sum_j A_{ij}\right)$$

b) $$\frac{d\vec{u}}{dt} = C \cdot (Au - Du)$$
$$\text{diag}(A\vec{1})$$

c) $$\frac{d\vec{u}}{dt} = C(A - D)u = -CLu$$

d) $$\frac{du}{dt} = 0 \Rightarrow -cLu = \vec{0} \qquad s = \frac{\sum_i u_i(0)}{n}$$
$$\vec{u} = s\vec{1}$$

Random walk

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a) $$p_i(t) = \sum_j \frac{A_{ij}}{k_j} p_j(t-1)$$

b) $$\vec{p}(t) = A D^{-1} p(t-1)$$

D^{-1}

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & & & & \\ 0 & & & & \\ 0 & & & & \\ 1 & & & & \end{bmatrix} \begin{bmatrix} 1/2 & & & & \\ & 1/2 & & & \\ & & 1/4 & & \\ & & & 1/5 & \\ & & & & 1/6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/4 & 0 & 1/6 \\ 1/2 & & & & \\ 0 & & & & \\ 0 & & & & \\ 1/2 & & & & \end{bmatrix}$$

c) $$P = A D^{-1} P$$

d) $p = AD^{-1}p \iff p - AD^{-1}p = \vec{0}$

$$(I - AD^{-1})\,p = \vec{0}$$

$$(D - A)\,D^{-1}p = \vec{0}$$

$$L\,\boxed{D^{-1}p} = \vec{0} \qquad \boxed{p_i = \frac{k_i}{2m}}$$

$D^{-1}p = \vec{1}$ $\quad / \cdot D$ (links)

$p = D\vec{1}$. $\qquad 1^T D 1 = 2m$

$$p = \frac{1}{2m} D\vec{1}$$

Normalized Graph Laplacian

$$\mathcal{L} = D^{-1/2} L D^{-1/2} \qquad \begin{bmatrix} 1/\sqrt{k_1} & & \\ & \ddots & \\ & & 1/\sqrt{k_n} \end{bmatrix}$$

$$\underline{P} = AD^{-1} = \underbrace{D^{1/2} (\underline{I} - \mathcal{L}) D^{-1/2}}$$

Excersice:

$$L = D - A$$

$$\mathcal{L} = \underline{I} - \underbrace{D^{-1/2} A D^{-1/2}}$$

share eigenvalues

$$M = S^{-1} N S$$

$P \qquad \underline{I} - \mathcal{L}$ are similar

$\lambda_n = 1 \qquad \lambda_1 = 0$

Katz centrality

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$$x_i = \alpha \sum_j A_{ij} x_j + \beta$$

$$x = \alpha A x + \beta \vec{1}$$

$$\alpha < \frac{1}{\lambda_1}$$

$$(x - \alpha A x) = \beta \vec{1}$$

$$(I - \alpha A) x = \beta \vec{1}$$

$$x = \beta (I - \alpha A)^{-1} \vec{1}$$

$$\Rightarrow \quad x = \boxed{(I - \alpha A)^{-1}} \vec{1} = \sum_{r=0}^{\infty} (\alpha A)^r \vec{1}$$

PageRank

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$$x = \alpha \boxed{AD^{-1}} x + \mathbb{1}$$

$\vdots$

Excercise

$$x = \left(I - \alpha \boxed{AD^{-1}}\right)^{-1} \mathbb{1}$$

$$\alpha < \frac{1}{\lambda_n} \Rightarrow \alpha < 1$$

$$\alpha = 0.85$$

$0.1 \quad 0.1^2 \quad 0.1^3 \quad 0.1^4$

α smaller iterate faster
but smaller precision

Inverse of matrix $O(n^3)$

Iteration $O(r \cdot m)$

Treshold τ $\alpha^r < \tau$

$\alpha^r = \tau$ $r \log \alpha = \log \tau$

$\tau = 10^{-6}$
$\alpha = 0.85$

$r = \frac{\log \tau}{\log \alpha} = \frac{-6}{\log(0.85)} \approx 30$

$10^{-8} \quad 10^{-9} \quad r \approx 100$

$\alpha = 0.85$ 100

David Gleich: Empiral Teleportation Factor

$\alpha = 0.5$ Wikipedia $\alpha = 0.4$

TUG Online

T4 44 $\alpha = 0.05$

Hubs & Authorities

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x authority score

y hub score

$$x_i = \alpha \sum_j A_{ij} y_j$$

$$y_i = \beta \sum_j A_{ji} x_j$$

$$x = \alpha A y \qquad y = \beta A^T x$$

$$x = \alpha A \beta A^T x = \alpha\beta A A^T x$$

$$\Leftrightarrow (AA^T) x = \frac{1}{\alpha\beta} x$$

$$\lambda_n = \frac{1}{\alpha\beta} \qquad AA^T$$

$y = \beta A^T x = \beta A^T \alpha A y = \alpha\beta A^T A y$

$x = \alpha A y$

$\frac{1}{\alpha\beta} y = (A^T A) y$

$\frac{1}{\alpha\beta} = \lambda_n$

$A^T A$ bibl. com.

$A A^T \quad A^T A$

Excers

Singular Value Decomp.

$A = U \Sigma V^T$

$A A^T$

$A^T A$

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$O(n+m)$ BFS

$O(n(n+m)) = O(n^2+nm) = O(nm)$

$O(n^2)$

$O(m) \geq O(n)$ $O(n^3)$

Sparse $O(m) \approx O(n)$

Dense $O(m) = O(n^2)$

Approximation: Centrality estimation in large Netw

$O(rm)$ Brandes $< 1\%$