Structured similarity

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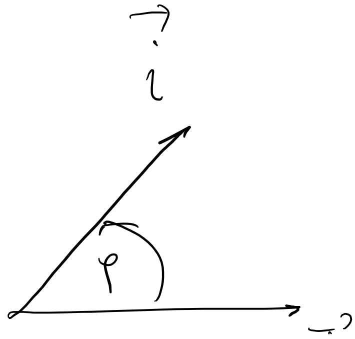
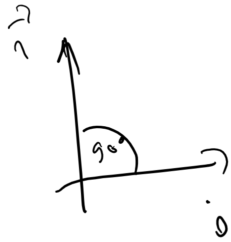
$$n_{ij} = \sum_k A_{ik} A_{kj}$$





$$\sigma_{ij} = \frac{n_{ij}}{\sqrt{k_i}\sqrt{k_j}} \qquad \cos\varphi = \frac{\vec{x}\,\vec{y}}{|x||y|}$$

$$\sqrt{\sum_j A_{ij}^2} = \sqrt{\sum_j A_{ij}} = \sqrt{k_i}$$

$h_{ij} \sim r_{ij}$

$h_{ij} = \sum_k A_{ik} A_{kj}$

$$\frac{k_i k_j}{n}$$

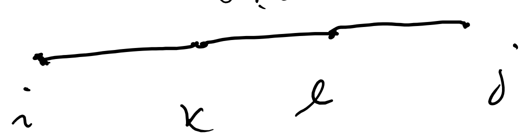
$$(X - E[X])(Y - E[Y])$$

$$\sigma_{ij} = n_{ij} - \frac{k_i k_j}{n} = \sum_k A_{ik} A_{kj} - \frac{k_i k_j}{n} - \frac{k_i k_j}{n} + \frac{k_i k_j}{n}$$

$$= \sum_k \left(A_{ik} A_{kj} - \frac{A_{ik} k_j}{n} - \frac{A_{kj} \cdot k_i}{n} + \frac{1}{n} \cdot \frac{k_i k_j}{n} \right)$$

$$= \sum_k \left(A_{ik} - \frac{k_i}{n} \right) \left(A_{kj} - \frac{k_j}{n} \right)^{n} = n \text{ times } \mathrm{cov}(A_i, A_j)$$

σ_{kl} is high



$$\delta_{ij} = \begin{cases} 1 & i \neq j \\ 0 & \text{oth.} \end{cases}$$

$$\sigma_{ij} = \alpha \sum_{kl} A_{ik} A_{jl} \sigma_{kl} + \delta_{ij}$$

$$\sigma = \alpha A \sigma A + I$$

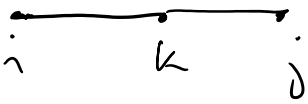
$$\sigma^0 = 0$$

$$\sigma^1 = I$$

$$\sigma^2 = \alpha A I A + I$$

$$= \alpha A^2 + I$$

$$\sigma^3 = \alpha^2 A^4 + \alpha A^2 + I$$



$\alpha < \frac{1}{\lambda_m}$

$$\sigma_{ij} = \alpha \sum_k A_{ik}\sigma_{kj} + \delta_{ij}$$

$$\sigma = \alpha A \sigma + I \qquad \sigma = \sum_{r=0}^{\infty} (\alpha A)^r$$

$$= (I - \alpha A)^{-1}$$

$$\sigma^0 = 0$$

$$\sigma^1 = I$$

$$\sigma^2 = \alpha A I + I = \alpha A + I$$

$$\sigma^3 = \alpha A(\alpha A + I) + I = \alpha^2 A^2 + \alpha A + I$$

$$\sigma = D^{-1}(I - \alpha A)^{-1} = D(D - \alpha A)^{-1}$$

Assortativity

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$$\delta(c_i, c_j) = \begin{cases} 1 & c_i = c_j \\ 0 & \text{other} \end{cases}$$

c_i is the group of node i

$$\frac{1}{2} \sum_{ij} A_{ij} \delta(c_i, c_j)$$

$$- \frac{1}{2} \sum_{ij} \frac{k_i k_j}{2m} \delta(c_i, c_j)$$

$$k_i \sim \frac{k_i k_j}{2m}$$

$$Q = \frac{1}{2} \sum_{ij} \underbrace{\left(A_{ij} - \frac{k_i k_j}{2m}\right)}_{B_{ij}} \delta(c_i, c_j)$$

modularity

B: modularity matrix

$$\ldots \quad Q_{max} = \frac{1}{2m} \left(2m - \sum_{ij} \frac{k_i k_j}{2m}\right)$$

Q / Q_{max}

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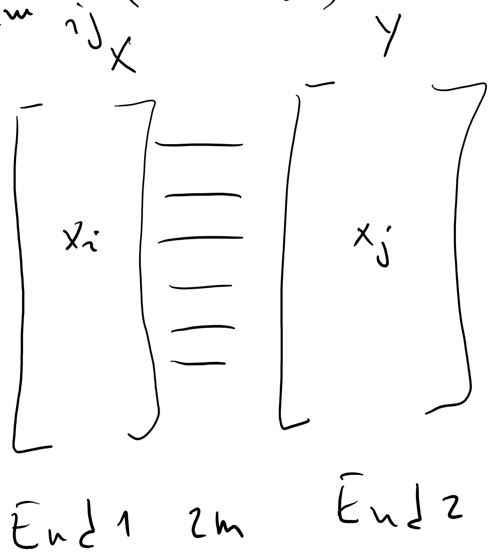
Assortivity for scalar values

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) x_i x_j$$



$$\mathrm{cov}(x,y) = \frac{\sum_{ij} A_{ij} (x_i - \mu)(x_j - \mu)}{\sum_{ij} A_{ij}}$$

$$A_{ij} = 1$$

$$= \frac{1}{2m} \sum_{ij} \overbrace{\left(A_{ij} - \frac{k_i k_j}{2m} \right)}^{B_{ij}} x_i x_j$$

μ = average

Newman Modularity

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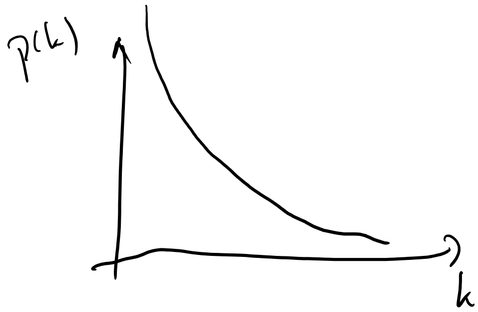
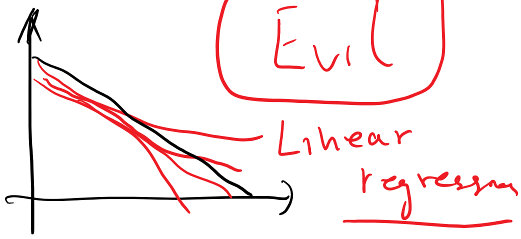
Degree Distr.



$p(k) \propto k^{-\alpha}$

$\boxed{\text{Evil}}$



1) Linear regression: independence is broken

Clauset: Power-Laws in Empirical Data log-normal

Power laws, Empirical Evidence, Clauset

Maxim. Likel. Estim.